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1982 J. Phys. A: Math. Gen. 15 3877

(http://iopscience.iop.org/0305-4470/15/12/036)

J. Phys. A: Math. Gen. **15** (1982) 3877–3879. Printed in Great Britain

Quantum electrodynamic charge space energy bands in singly connected superconducting weak links

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Received 5 May 1982

Abstract. In a weak-link constriction between two bulk superconductors it is well known that the condensed matter Hamiltonian exhibits periodicity in magnetic flux space. Here it is shown that a quantum electrodynamic treatment of the voltage across the weak link yields energy bands periodic in the space of Maxwell electric flux displacement charge.

In a purely spatial gauge (Ginzburg 1979), the quantum electric field operator can be written as

$$\boldsymbol{E} = (\partial \boldsymbol{A}/c\,\partial t) = -(\mathrm{i}/\hbar)[\mathcal{H}, \boldsymbol{A}]. \tag{1}$$

To observe quantum electrodynamic effects on a macroscopic length scale it is convenient to describe the electromagnetic field modes in terms of electrical engineering circuits (Widom 1979). The voltage across a circuit element (in quantum electrodynamics) uses equation (1) written in terms of Faraday's law

$$V = -(\mathrm{d}\Phi/c\ \mathrm{d}t) = -(\mathrm{i}/\hbar c)[\mathcal{H}, \Phi]. \tag{2}$$

For a circuit mode with a geometrical capacitance C, the quantum electrodynamic Hamiltonian reads

$$\mathcal{H} = -(\hbar^2 c^2/2C)(\partial/\partial\Phi)^2 + H(\Phi) \tag{3}$$

where $H(\Phi)$ represents the condensed matter Hamiltonian as a function of magnetic flux Φ, and the first term on the right-hand side of equation (3) represents capacitive electric field energy storage.

Quantum electrodynamic circuit Hamiltonians, as in equation (3), are by no means restricted to superconducting circuits. However, for a superconducting weak-link constriction between two bulk superconductors, the classification of energy levels is especially interesting. In particular, energy bands periodic in the space of Maxwell displacement charge will be theoretically shown to exist. Previously introduced notions of a current frequency bias (Widom and Clark 1982) and a critical voltage law (Widom and Clark 1980), which represent quantum electrodynamic duality transformations of the usual Josephson effects, will here be shown to be simple consequences of the Hamiltonian symmetry. The observations of such duality would constitute further evidence (Prance *et al* 1981) of the validity of quantum electrodynamics on macroscopic length scales.

0305-4470/82/123877+03$02.00

The distinguishing feature of the singly connected weak-link condensed matter Hamiltonian is the effective flux space periodicity (Anderson 1964)

$$H(\Phi+\Phi_0)=H(\Phi) \tag{4}$$

where the flux quantum

$$\Phi_0=(\pi\hbar c/e) \tag{5}$$

derives its magnitude from the charge of the carriers of the superconducting current

$$q=2e. \tag{6}$$

From equations (3)–(6), it is evident that the energy eigenstates

$$\mathscr{H}\psi_{nQ}(\Phi,\beta)=E_n(Q)\psi_{nQ}(\Phi,\beta) \tag{7}$$

where β represents condensed matter degrees of freedom, can be classified according to the Bloch theorem

$$\psi_{nQ}(\Phi+\Phi_0,\beta)=\exp(-\mathrm{i}Q\Phi_0/\hbar c)\psi_{nQ}(\Phi,\beta) \tag{8}$$

with the energy band periodicity condition

$$E_n(Q+q)=E_n(Q). \tag{9}$$

In that the voltage, see equation (2), is by Faraday's law a formal 'velocity in flux space', it is evident that the 'group velocity' in Maxwell displacement charge space (Callaway 1976a)

$$v_n(Q)=\mathrm{d}E_n(Q)/\mathrm{d}Q \tag{10}$$

represents the mean voltage in state ψ_{nQ}.

For a weak link in thermal equilibrium, the free energy

$$f(Q,T)=-k_\mathrm{B}T\ln\sum_n\exp(-E_n(Q)/k_\mathrm{B}T) \tag{11}$$

determines the mean voltage equation of state $v(Q,T)$ via the thermodynamic law

$$\mathrm{d}f=-s\,\mathrm{d}T+v\,\mathrm{d}Q. \tag{12}$$

Equations (9), (11) and time reversal symmetry dictate a harmonic expansion of the form

$$f(Q,T)=f_0(T)-\hbar\sum_{l=1}^{\infty}\Omega_l(T)\cos(2\pi lQ/q). \tag{13}$$

The critical voltage law results from equations (12) and (13); it is

$$v(Q,T)=(\Phi_0/c)\sum_{l=1}^{\infty}l\Omega_l(T)\sin(2\pi lQ/q) \tag{14}$$

where $\Omega_l(T)$ is the thermal mean frequency of magnetic flux tunnelling events carrying $l\Phi_0$ flux units. If a current source, uniform in time, were placed in series with the weak link and if the voltage followed the charge (Callaway 1976b) $(\mathrm{d}Q/\mathrm{d}t)=I$ in a quasistatic fashion, then it would follow that the voltage would oscillate with a bias frequency

$$\omega_I=(\pi I/e). \tag{15}$$

In the more general situation of a current source where

$$(\mathrm{d}Q/\mathrm{d}t) = I - J\cos(\omega t) \tag{16}$$

equations (14) and (16) would yield a voltage oscillation of the form

$$v(t) = (\Phi_0/c) \sum_{l=1}^{\infty} \sum_{n=-\infty}^{+\infty} l\Omega_l J_n(l\omega_J/\omega) \sin(l\theta + l\omega_I t - n\omega t) \tag{17}$$

where $\omega_J = \pi J/e$ and $J_n(x)$ is the Bessel function of order n. If equation (17) were compared with the usual Josephson voltage bias frequency effects (Tilley and Tilley 1974), then it would become evident that the important feature is that a voltage component uniform in time exists when $l\omega_I = n\omega$. From a quantum electrodynamic viewpoint, this frequency condition can be written in terms of the energy,

$$\Delta E = l(I\Phi_0/c) = n\hbar\omega \tag{18}$$

i.e. n photons from the periodic part of the current source can induce l Faraday–London quantum bundles of magnetic flux to pass across the weak-link construction. This requires $(I\Phi_0/c)$ worth of work per fluxoid against that part I of the current source uniform in time.

For practical values of I, J and ω, it would appear that power absorption in the weak link, as in equation (18), due to the conversion of photon radiation into fluxoids (and vice versa) requires about $n \sim 10^4$ photons. For such large n it would also appear that radiofrequency photon pumping spectroscopy (Novikov and Skrotskii 1978) might be the required experimental tool for observations of equation (18).

Quantum electrodynamics thus provides a theoretical classification of the energy bands in Maxwell displacement charge space although, as yet, these effects have not been observed in singly connected superconducting weak links.

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